

# Symmetry Nonrestoration in a Gross-Neveu Model with Random Chemical Potential

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## Abstract

We study the symmetry behavior of the Gross-Neveu model in three and two dimensions with random chemical potential. This is equivalent to a four-fermion model with charge conjugation symmetry as well as  $Z_2$  chiral symmetry. At high temperature the  $Z_2$  chiral symmetry is always restored. In three dimensions the initially broken charge conjugation symmetry is not restored at high temperature, irrespective of the value of the disorder strength. In two dimensions and at zero temperature the charge conjugation symmetry undergoes a quantum phase transition from a symmetric state (for weak disorder) to a broken state (for strong disorder) as the disorder strength is varied. For any given value of disorder strength, the high-temperature behavior of the charge conjugation symmetry is the same as its zero-temperature behavior. Therefore, in two dimensions and for strong disorder strength the charge conjugation symmetry is not restored at high temperature.

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## I. INTRODUCTION

Intuitively, when heated, a system with initially broken symmetry will recover its symmetry because thermal fluctuations are able to overcome potential barriers. But a counterexample was noticed by Weinberg [1]: For the four-dimensional  $O(N)\times O(N)$  scalar  $\phi^4$  model, he showed that the system can remain in the broken phase even at sufficiently high temperature. This phenomenon is called inverse symmetry breaking or symmetry nonrestoration (SNR), depending on whether the system was in a symmetric or a broken phase at zero temperature.

Since Weinberg's observation, SNR has been a subject of academic curiosity or a candidate way out of cosmological problems caused by topological defects like monopoles and domain walls (see Ref. [2] for review). According to Bajc's classification [2], there are three classes of SNR mechanisms in field theory: (i) a prototype case like the two-scalar model [1,3], (ii) flat directions in supersymmetric theories [4], and (iii) large charge density (or chemical potential) [5,6]. Here we restrict ourselves to class (iii).

If a large enough charge can not be stored in thermally excited modes at high temperature, it must reside in the vacuum, and this is a sign of SNR. In field theory, a scalar field (order-parameter field) gets a positive mass term by thermal effects, but a negative one by the effects due to the chemical potential. For a fixed charge (i.e., in the canonical formalism), the chemical potential is temperature dependent. In this case, if the effect of the chemical potential on the mass exceeds the thermal effects at sufficiently high temperature, the scalar field acquires a nonzero vacuum expectation value (i.e., SNR) [5].

However, in an open system of the model which does not belong to the class (i) or (ii), the symmetry may always be restored at high temperature. In the grand canonical formalism, the chemical potential and the temperature are independent parameters and so the thermal effect on the mass always surpasses the effect of the chemical potential at sufficiently high temperature, for a fixed chemical potential. For example, consider the Gross-Neveu (GN) model [7–9] with chiral symmetry. At finite chemical potential the initially broken chiral

symmetry is always restored at high temperature [10].

In order to find a new kind of SNR in four-fermion models, we will extend the GN model at finite chemical potential to a disordered model with random chemical potential. Recently, disordered nonrelativistic Dirac fermions in two spatial dimensions have been studied in relation to the integer quantum Hall transition [11]. Pure fermions exhibit such a transition as the value of the mass is tuned through zero, but its universality class is different from the one observed in actual experiments. Usually three types of (static) disorder are considered for a more realistic model: random gauge potential, random chemical potential and random mass.

Motivated by the SNR mechanism (iii), we introduce the (relativistic) GN model with random chemical potential in Sec. II. If the chemical potential has a Gaussian distribution at each site, our model is equivalent to the four-fermion model with two kinds of four-fermion interaction,  $(\bar{\Psi}\Psi)^2$  and  $(\bar{\Psi}\gamma_0\Psi)^2$  (see Eq.(3)), and has charge conjugation symmetry in addition to the  $Z_2$  chiral symmetry. In Sec. III we examine the behavior of these symmetries as the temperature or disorder strength is varied using the  $1/N$  expansion in three and two dimensions. While  $Z_2$  chiral symmetry is always restored at high temperature, the charge conjugation symmetry exhibits SNR. In addition, we check the validity of the mean field approximation (the leading approximation in the  $1/N$  expansion) in two dimensions. In Sec. IV the fundamental origin of SNR for charge conjugation symmetry is discussed conceptually. Our conclusions are presented in Sec. V.

## II. GROSS-NEVEU MODEL WITH RANDOM CHEMICAL POTENTIAL

The Euclidean Lagrangian of the GN model at finite chemical potential  $\mu$  is given by

$$\mathcal{L} = \bar{\Psi}(\not{\partial} + \mu\gamma_0)\Psi - \frac{g^2}{2N}(\bar{\Psi}\Psi)^2, \quad (1)$$

where  $g^2(> 0)$  is the coupling constant of the four-fermion interaction  $(\bar{\Psi}\Psi)^2$  and  $N$  is the number of flavors of the Dirac fermion  $\Psi$ . The  $\gamma$  matrices are  $4 \times 4$  and hermitian.

Let us consider the system under the influence of a random chemical potential  $\rho(x)$  with the Gaussian distribution  $\exp(-\int d^d x \frac{N}{2R^2} \rho^2)$  at each site where  $R^2(> 0)$  is the strength of disorder and  $d$  the dimension of the Euclidean space. The Gaussian noise [12] is characterized by correlation functions

$$\langle \rho(x) \rangle = 0, \quad \langle \rho(x) \rho(x') \rangle = \frac{R^2}{N} \delta^d(x - x'). \quad (2)$$

After integrating out the random chemical potential, our model is equivalent to the four-fermion model

$$\mathcal{L} = \bar{\Psi} \not{\partial} \Psi - \frac{1}{2N} \left[ g^2 (\bar{\Psi} \Psi)^2 + R^2 (\bar{\Psi} \gamma_0 \Psi)^2 \right], \quad (3)$$

with the  $Z_2$  chiral symmetry  $\{\Psi \rightarrow \gamma_5 \Psi, \bar{\Psi} \rightarrow -\bar{\Psi} \gamma_5\}$  and the charge conjugation symmetry  $\{\Psi \rightarrow C \bar{\Psi}^T, \bar{\Psi} \rightarrow -\Psi^T C^\dagger\}$ . Here the matrix  $C$  satisfies  $C^\dagger C = 1$ ,  $C^\dagger \gamma_\mu C = -\gamma_\mu^T$ . Under charge conjugation,  $\bar{\Psi} \Psi$  and  $\bar{\Psi} \gamma_0 \Psi$  transform to  $\bar{\Psi} \Psi$  and  $-\bar{\Psi} \gamma_0 \Psi$  respectively. Hence, the Lagrangian Eq.(1) with definite chemical potential  $\mu$  does not possess the charge conjugation symmetry (i.e., fermion-antifermion symmetry). Note that in Eq.(3) the chemical potential term does not appear explicitly.

We will study this model by the leading approximation of the  $1/N$  expansion in three and two dimensions. To easily incorporate the  $1/N$  expansion, let us rewrite the Lagrangian Eq.(3) by introducing scalar auxiliary fields  $\sigma(x)$  and  $\rho(x)$ :

$$\mathcal{L} = \bar{\Psi} (\not{\partial} + \sigma + \rho \gamma_0) \Psi + \frac{N}{2g^2} \sigma^2 + \frac{N}{2R^2} \rho^2. \quad (4)$$

The random chemical potential  $\rho(x)$  plays the role of a scalar auxiliary field. The  $Z_2$  chiral symmetry and the charge conjugation symmetry are now expressed as  $\{\Psi \rightarrow \gamma_5 \Psi, \bar{\Psi} \rightarrow -\bar{\Psi} \gamma_5, \sigma \rightarrow -\sigma\}$  and  $\{\Psi \rightarrow C \bar{\Psi}^T, \bar{\Psi} \rightarrow -\Psi^T C^\dagger, \rho \rightarrow -\rho\}$ , respectively.

### III. THE BEHAVIOR OF $Z_2$ CHIRAL SYMMETRY AND CHARGE CONJUGATION SYMMETRY AT ZERO AND HIGH TEMPERATURE

For finite-temperature field theory we adopt the imaginary-time formalism. At inverse temperature  $\beta(= T^{-1})$ , the fermion fields are antiperiodic on  $R^{d-1} \times [0, \beta]$ , while the scalar

auxiliary fields are periodic. Let us introduce the notation:  $\int_p^{(T)} \equiv T \sum_{n=-\infty}^{\infty} \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}}$ ,  $\int_p^{(0)} \equiv \int \frac{d^d p}{(2\pi)^d}$ . Integrating out the fermion fields in the partition function for Eq.(4) we obtain the effective action for the auxiliary fields  $\sigma$  and  $\rho$ . In order to investigate the vacuum structure we need to find the finite-temperature effective potential  $V_T(\sigma, \rho)$  by taking  $\sigma$  and  $\rho$  as constant fields: To leading order in the  $1/N$  expansion,

$$\frac{V_T(\sigma, \rho)}{N} = \frac{\sigma^2}{2g^2} + \frac{\rho^2}{2R^2} - 2 \int_p^{(T)} \ln[(p_0 - i\rho)^2 + \mathbf{p}^2 + \sigma^2], \quad (5)$$

where  $p_0 = (2n+1)\pi/\beta \equiv \omega_n$  ( $n = \text{integer}$ ) at nonzero temperature. Note that the effect of the chemical potential  $\rho$  is to shift the energy by  $-i\rho$ .

To evaluate the integration in Eq.(5), we need some mathematical formulae. By the standard method of contour integration [13],

$$T \sum_{n=-\infty}^{\infty} \frac{1}{(\omega_n - i\rho)^2 + \sigma^2} = \frac{1}{2|\sigma|} \left[ 1 - \frac{1}{1 + e^{\beta(|\sigma| + |\rho|)}} - \frac{1}{1 + e^{\beta(|\sigma| - |\rho|)}} \right], \quad (6)$$

$$\begin{aligned} T \sum_{n=-\infty}^{\infty} \frac{\omega_n - i\rho}{(\omega_n - i\rho)^2 + \sigma^2} &= i\rho T \sum_{n=-\infty}^{\infty} \frac{\omega_n^2 - (\sigma^2 - \rho^2)}{(\omega_n^2 + \sigma^2 - \rho^2)^2 + (2\rho\omega_n)^2} \\ &= \frac{i}{2} \left[ \frac{\sinh(\beta\rho)}{\cosh(\beta\sigma) + \cosh(\beta\rho)} \right]. \end{aligned} \quad (7)$$

When the GN model is studied in the canonical formalism (i.e., with a fixed charge) [14], similar calculations appear with imaginary chemical potential. In this case a regulating factor of the form  $e^{i\omega_n\tau}$  is needed in evaluating the summation in Eq.(7) and ensures a finite result in the limit  $\tau \rightarrow 0$  after the Matsubara sum has been performed. By using Eqs.(6) and (7), we obtain

$$T \sum_{n=-\infty}^{\infty} \ln[(\omega_n - i\rho)^2 + \sigma^2] = T [\ln 2 + \ln(\cosh(\beta\sigma) + \cosh(\beta\rho))], \quad (8)$$

where the  $\zeta$ -function regularization was used to determine the field-independent constant.

At zero temperature, these formulae reduce to

$$\int \frac{dp_0}{2\pi} \frac{1}{(p_0 - i\rho)^2 + \sigma^2} = \frac{\theta(|\sigma| - |\rho|)}{2|\sigma|}, \quad (9)$$

$$\int \frac{dp_0}{2\pi} \frac{p_0 - i\rho}{(p_0 - i\rho)^2 + \sigma^2} = \frac{i}{2} \text{sgn}(\rho) \theta(|\rho| - |\sigma|), \quad (10)$$

$$\int \frac{dp_0}{2\pi} \ln[(p_0 - i\rho)^2 + \sigma^2] = \max(|\sigma|, |\rho|). \quad (11)$$

Using Eqs.(6), (7) and  $E_\sigma \equiv \sqrt{\mathbf{p}^2 + \sigma^2}$ , we have

$$\frac{\partial}{\partial \sigma} \left( \frac{V_T}{N} \right) = \frac{\sigma}{g^2} - 2\sigma \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \frac{1}{E_\sigma} \left[ 1 - \frac{1}{1 + e^{\beta(E_\sigma + \rho)}} - \frac{1}{1 + e^{\beta(E_\sigma - \rho)}} \right], \quad (12)$$

$$\frac{\partial}{\partial \rho} \left( \frac{V_T}{N} \right) = \frac{\rho}{R^2} - 2 \sinh(\beta\rho) \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \frac{1}{\cosh(\beta E_\sigma) + \cosh(\beta\rho)}. \quad (13)$$

To renormalize the effective potential  $V_T(\sigma, \rho)$  let us consider the GN model at zero temperature and in the absence of the random chemical potential because the effects of temperature and chemical potential do not change the ultraviolet behavior. Define  $1/G^2 \equiv 1/g^2 - 1/g_c^2$  with  $1/g_c^2 \equiv 4 \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2}$ . In  $2 < d < 4$ , the GN model is in the broken phase of the  $Z_2$  chiral symmetry for negative  $G^2$ , corresponding to strong coupling ( $g^2 > g_c^2$ ) in the cutoff regularization, while it is in the symmetric phase for  $G^2 \geq 0$ , corresponding to weak coupling ( $0 < g^2 \leq g_c^2$ ). In particular, in two dimensions, the  $Z_2$  chiral symmetry of the GN model must be broken no matter how we choose the coupling  $g^2$  [8]. In the broken phase,

$$\frac{1}{g^2} = 4 \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 + M^2}, \quad (14)$$

where  $M = |\langle \sigma \rangle| (> 0)$  is the dynamically generated fermion mass at zero temperature.

From now on, we will adopt dimensional regularization, where  $G^2$  is equal to the regularized  $g^2$ .

### A. Three dimensions

In this case, renormalization is not needed to the leading order of the  $1/N$  expansion (in dimensional regularization). By making use of Eq.(11), we can find the zero-temperature effective potential  $V_0(\sigma, \rho)$  directly:

$$\frac{V_0(\sigma, \rho)}{N} = \frac{\sigma^2}{2G^2} + \frac{\rho^2}{2R^2} - \frac{1}{6\pi} \left[ \max^3(|\sigma|, |\rho|) - 3\sigma^2 \max(|\sigma|, |\rho|) \right], \quad (15)$$

where  $1/G^2 = -M/\pi$  for broken  $Z_2$  chiral symmetry. The gap equations have four kinds of solution  $(|\sigma|, |\rho|)$ : (i)  $(0, 0)$ , (ii)  $(M, 0)$ , (iii)  $(0, 2\pi/R^2)$  and (iv)  $(\sqrt{M(M - 2\pi/R^2)}, M)$ . The solution (iv) exists only for  $M > 2\pi/R^2$  and corresponds to saddle points. Fig. 1 shows

the zero-temperature effective potential as a function of  $\sigma/M$  and  $\rho/M$ , for broken  $Z_2$  chiral symmetry.  $\langle \rho \rangle = 0$  is metastable, irrespective of the values of  $G^2$  and  $R^2$ . For  $|\rho| > |\sigma|$ , however,  $V_0(\sigma, \rho)$  is unbounded from below due to the  $-|\rho|^3/(6\pi)$  term, which indicates breaking of the charge conjugation symmetry. This result stems from the fact that the term  $-|\rho|^3/(6\pi)$  arising from quantum effects surpasses the effect of the probability distribution  $(\rho^2/(2R^2))$  for large  $|\rho|$ .

At finite temperature, using Eqs.(6), (7) and dimensional regularization, we obtain

$$\int_p^{(T)} \frac{1}{(\omega_n - i\rho)^2 + E_\sigma^2} = -\frac{1}{4\pi\beta} \left[ \beta|\sigma| + \ln \left( 1 + 2e^{-\beta|\sigma|} \cosh(\beta\rho) + e^{-2\beta|\sigma|} \right) \right], \quad (16)$$

$$\int_p^{(T)} \frac{\omega_n - i\rho}{(\omega_n - i\rho)^2 + E_\sigma^2} = -\frac{i \operatorname{sgn}(\rho)}{4\pi\beta^2} \left[ \beta|\sigma| \ln \left( \frac{1 + e^{\beta(|\sigma|+|\rho|)}}{1 + e^{\beta(|\sigma|-|\rho|)}} \right) + \operatorname{Li}_2(-e^{\beta(|\sigma|+|\rho|)}) - \operatorname{Li}_2(-e^{\beta(|\sigma|-|\rho|)}) \right]. \quad (17)$$

Here the polylogarithm  $\operatorname{Li}_\nu(z)$  is defined (for  $\nu > 0$ ) as  $\operatorname{Li}_\nu(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\nu}$  (see Ref. [15] for useful properties). From these formulae, the finite-temperature effective potential  $V_T(\sigma, \rho)$  is given by

$$\begin{aligned} \frac{V_T(\sigma, \rho)}{N} = & \frac{\sigma^2}{2G^2} + \frac{\rho^2}{2R^2} - \frac{\sigma^3}{3\pi} + \frac{1}{\pi\beta^3} \left[ \operatorname{Li}_3(-e^{\beta(\sigma+\rho)}) + \operatorname{Li}_3(-e^{\beta(\sigma-\rho)}) \right. \\ & \left. - \beta\sigma \left\{ \operatorname{Li}_2(-e^{\beta(\sigma+\rho)}) + \operatorname{Li}_2(-e^{\beta(\sigma-\rho)}) \right\} \right], \end{aligned} \quad (18)$$

up to a field-independent constant. At sufficiently high temperature,

$$\frac{V_T(\sigma, \rho)}{N} \approx \left( \frac{\ln 2}{\pi} \right) T(\sigma^2 - \rho^2). \quad (19)$$

While the initially broken  $Z_2$  chiral symmetry is restored at high temperature, charge conjugation symmetry is not. Hence our model exhibits nonrestoration of charge conjugation symmetry irrespective of the values of  $G^2$  and  $R^2$ . We may interpret this phenomenon as an inverse symmetry breaking because  $\langle \rho \rangle = 0$  is metastable at zero temperature. Intuitively, SNR is related to the tachyon-like behavior of the random chemical potential (see Eqs.(19) and (24)). In the quantum correction term of Eq.(5) the chemical potential acts as a negative mass term  $(-\rho^2)$  contrary to the usual positive mass term  $(\sigma^2)$ . From a different point of view, we will discuss the origin of SNR conceptually in Sec. IV.

## B. Two dimensions

For dimensional regularization, we work in  $2 + \epsilon$  dimensions. In terms of the fermion mass  $M$ , the zero-temperature effective potential is given by

$$\begin{aligned} \frac{V_0(\sigma, \rho)}{N} = & \frac{\sigma^2}{2\pi} \left[ -1 + 2 \ln \left( \frac{\max(|\sigma|, |\rho|) + \sqrt{\max^2(|\sigma|, |\rho|) - \sigma^2}}{M} \right) \right] \\ & + \frac{\rho^2}{2R^2} - \frac{|\rho|}{\pi} \sqrt{\max^2(|\sigma|, |\rho|) - \sigma^2}, \end{aligned} \quad (20)$$

where Eqs.(11) and (14) were used. Unlike in three dimensions, for large  $|\rho|$  the effect of the probability distribution ( $\rho^2/(2R^2)$ ) is comparable to the last term ( $\approx -\rho^2/\pi$ ) in Eq.(20) arising from quantum effects. The charge conjugation symmetry can be controlled by the strength of disorder  $R^2$ . The system is in the symmetric state for  $0 < R^2 < \pi/2$ , while in the broken state for  $R^2 > \pi/2$ . Fermions and antifermions are equally probable in the symmetric state ( $\langle \rho \rangle = 0$ ), but only fermions (or antifermions) are allowed in the broken state ( $\langle \rho \rangle = \pm\infty$ ). Our system suffers from a quantum phase transition at  $R^2 = \pi/2 (\equiv R_c^2)$ . The gap equations have solutions ( $|\sigma|, |\rho|$ ): (i)  $(0, 0)$ , (ii)  $(M, 0)$  and (iii)  $(\sqrt{(2R^2 - \pi)/(2R^2 + \pi)}M, 2R^2M/(2R^2 + \pi))$  for  $R^2 \neq R_c^2$ , and (i)  $(0, \forall |\rho|)$  and (ii)  $(M, 0)$  for  $R^2 = R_c^2$ . The solution (iii) exists only for  $R^2 > R_c^2$  and corresponds to saddle points. Fig. 2 shows the zero-temperature effective potential as a function of  $\sigma/M$  and  $\rho/M$  in (a) the symmetric and (b) the broken phase for the charge conjugation symmetry.

To examine the high-temperature ( $\beta \rightarrow 0$ ) behavior, let us introduce dimensionless quantities:  $\tilde{V}_T = \beta^2 V_T$ ,  $\tilde{\sigma} = \beta\sigma$ ,  $\tilde{\rho} = \beta\rho$ ,  $\tilde{M} = \beta M$ . We want to expand the finite-temperature effective potential in  $\tilde{\sigma}$  and  $\tilde{\rho}$  at high temperature (i.e., for small  $\tilde{\sigma}$  and  $\tilde{\rho}$ ). In terms of the dimensionless quantities, Eqs.(12) and (13) are reduced to

$$\begin{aligned} \frac{\partial}{\partial \tilde{\sigma}} \left( \frac{\tilde{V}_T}{N} \right) &= \frac{2\tilde{\sigma}}{\pi} \left[ \ln \left( \frac{|\tilde{\sigma}|}{\tilde{M}} \right) + \int_0^\infty dx \frac{1}{\sqrt{x^2 + \tilde{\sigma}^2}} \left( \frac{1}{1 + e^{\sqrt{x^2 + \tilde{\sigma}^2} + \tilde{\rho}}} + \frac{1}{1 + e^{\sqrt{x^2 + \tilde{\sigma}^2} - \tilde{\rho}}} \right) \right] \\ &= \frac{2\tilde{\sigma}}{\pi} \left[ \left\{ \ln \left( \frac{\pi}{\tilde{M}} \right) - \gamma + O(\tilde{\sigma}^2) \right\} + \left\{ \frac{7\zeta(3)}{4\pi^2} + O(\tilde{\sigma}^2) \right\} \tilde{\rho}^2 + O(\tilde{\rho}^4) \right], \quad (21) \\ \frac{\partial}{\partial \tilde{\rho}} \left( \frac{\tilde{V}_T}{N} \right) &= \tilde{\rho} \left[ \frac{1}{R^2} - \frac{2 \sinh(\tilde{\rho})}{\pi \tilde{\rho}} \int_0^\infty dx \frac{1}{\cosh(\sqrt{x^2 + \tilde{\sigma}^2}) + \cosh(\tilde{\rho})} \right] \end{aligned}$$



$$= \tilde{\rho} \left[ \frac{1}{R^2} - \frac{2}{\pi} + \left\{ \frac{7\zeta(3)}{2\pi^3} + \mathcal{O}(\tilde{\rho}^2) \right\} \tilde{\sigma}^2 + \mathcal{O}(\tilde{\sigma}^4) \right]. \quad (22)$$

To obtain the  $\mathcal{O}(\tilde{\rho}^0)$  term in the bracket of Eq.(21) we used the integration formula [16] for small  $\tilde{\sigma}^2$ :

$$\int_0^\infty dx \frac{1}{\sqrt{x^2 + \tilde{\sigma}^2} (1 + e^{\sqrt{x^2 + \tilde{\sigma}^2}})} = -\frac{1}{2} \left[ \ln \left( \frac{|\tilde{\sigma}|}{\pi} \right) + \gamma + \mathcal{O}(\tilde{\sigma}^2) \right]. \quad (23)$$

Integrating Eqs.(21) and (22), at sufficiently high temperature, we obtain

$$\frac{V_T(\sigma, \rho)}{N} = \frac{1}{\pi} \left[ \ln \left( \frac{\pi T}{M} \right) - \gamma \right] \sigma^2 + \frac{1}{2} \left( \frac{1}{R^2} - \frac{2}{\pi} \right) \rho^2 + \frac{7\zeta(3)}{4\pi^3 T^2} \sigma^2 \rho^2 + \dots, \quad (24)$$

up to a field-independent constant. Since the  $\mathcal{O}(\tilde{\sigma}^0)$  term in the bracket of Eq.(22) is exact,  $V_T(\sigma, \rho)$  has no term higher than the  $\rho^2$  term that consists of  $\rho$  fields only. At high temperature, the  $Z_2$  chiral symmetry is always restored. However, the behavior of the charge conjugation symmetry is the same as that at zero temperature. Hence, charge conjugation symmetry is not restored at high temperature, for  $R^2 > R_c^2$ .

Until now in order to find the effective potential  $V_T(\sigma, \rho)$  we have used the mean field approximation (MFA) by taking  $\sigma$  and  $\rho$  as constant fields. This corresponds to the leading approximation in the  $1/N$  expansion where the  $\sigma$  and  $\rho$  loops (i.e., the fluctuations of the  $\sigma$  and  $\rho$  fields) are not included. Let us check the validity of our calculations. In the large  $N$  limit MFA is good, while for large but finite  $N$  it may fail due to the contribution from kinks in two dimensions [17,18].

At first consider the case of the usual two-dimensional GN model (without the random chemical potential) where MFA predicts a wrong critical temperature  $T_0 (\neq 0)$  [17]. For  $0 \leq T < T_0$  the MFA effective potential is the double well with two degenerate minima at  $M(T)$  and  $-M(T)$ , the solutions of the gap equation. Due to quantum tunneling between two degenerate minima, the system has kink solutions alternating between  $M(T)$  and  $-M(T)$ . They have higher energies than the constant solutions  $M(T)$  and  $-M(T)$ . The Helmholtz free energy  $F$  is related to the internal energy  $U$  and the entropy  $S$  by  $F = U - TS$ . Since  $F = U$  at zero temperature, the constant solution is favored and MFA is expected to be

valid. If we consider the contribution from kinks explicitly, we find the average (or blocking) potential [19] which has a plateau between  $M(\equiv M(0))$  and  $-M$  [17]. Even in the presence of an arbitrarily small external field this potential is tilted to favor  $M$  or  $-M$ . Hence the contribution from kinks does not change the physics materially and MFA is qualitatively valid at zero temperature. For  $0 < T < T_0$  the number of kink configurations is sufficiently large to gain enough entropy and so their probability is overwhelming. Since the region of  $\sigma = M(T)$  will, on the average, have the same weight as those of  $\sigma = -M(T)$ , we have  $\langle \sigma \rangle = 0$ , which indicates the breakdown of MFA. For  $T \geq T_0$  the system is in the symmetric phase ( $M(T) = 0$ ) in MFA and thus has no kink solutions. Consequently, for the two-dimensional GN model MFA is good only at zero and high temperature ( $T \geq T_0$ ) and, by the formation of kinks, the true critical temperature turns out to be zero.

Now let us examine the validity of MFA for the two-dimensional GN model with the random chemical potential Eqs.(3) or (4). For the purpose of the present paper we consider only the cases of zero and sufficiently high temperature. At zero temperature the MFA effective potential  $V_0(\sigma, \rho)$  has degenerate minima at  $(|\sigma|, |\rho|)$ :  $(M, 0)$  for  $0 < R^2 \leq R_c^2$  and  $(0, \infty)$  for  $R^2 > R_c^2$  (see Fig. 2 (a) and (b)). Hence for  $0 < R^2 \leq R_c^2$  our system may have kink solutions for  $\sigma$  alternating between  $M$  and  $-M$ , but no kinks for  $\rho$ . In this case the situation is similar to that of the usual GN model in the previous paragraph. Thus it is expected that MFA is good at zero temperature and for  $0 < R^2 \leq R_c^2$ . For  $R^2 > R_c^2$  the MFA effective potential is unbounded from below and there is no tunneling between two degenerate ground states because of infinitely high barrier. So  $\sigma$  and  $\rho$  do not have kink solutions. At sufficiently high temperature the  $\sigma$  and  $\rho$  fields are decoupled from each other in  $V_T(\sigma, \rho)$  and can be treated separately. By the restoration of the  $Z_2$  chiral symmetry in MFA,  $\sigma$  has no kink solutions, irrespective of the value of the disorder strength. For  $0 < R^2 < R_c^2$  the charge conjugation symmetry is preserved in MFA and so no kinks for  $\rho$ . For  $R^2 > R_c^2$  the situation is the same as that at zero temperature. For  $R^2 = R_c^2$  the MFA effective potential for  $\rho$  vanishes. As  $R^2$  is tuned through  $R_c^2$ , the charge conjugation symmetry undergoes a first-order phase transition from the symmetric phase ( $\langle \rho \rangle = 0$ ) to

the broken phase ( $\langle \rho \rangle = \infty$  or  $-\infty$ ), following positive or negative values of  $\langle \rho \rangle$  according to the value of  $\langle \rho \rangle$  in the broken phase. So it is reasonable to assume that  $\rho$  has no kink solution at  $R^2 = R_c^2$ . As a result, MFA is reliable at zero and high temperature for all values of the disorder strength.

#### IV. ORIGIN OF CHARGE CONJUGATION SYMMETRY NONRESTORATION

In this section we discuss the mechanism of SNR for the charge conjugation symmetry conceptually. For convenience set  $\sigma = 0$  in Eqs.(4) and (5) because SNR is the effect of the random chemical potential. That is, we neglect the four-fermion interaction  $(\bar{\Psi}\Psi)^2$  and consider the system of free massless Dirac fermions in the presence of the random chemical potential. To leading order in the  $1/N$  expansion the finite-temperature effective potential  $V_T(0, \rho)/N$  consists of two parts: (i) a term from probability distribution ( $\rho^2/(2R^2) \equiv P_R(\rho)$ ) and (ii) the grand free energy (or grand potential) for free massless Dirac fermions at constant chemical potential  $\rho$  ( $\equiv \Omega_T(\rho)$ ).

So it is essential to conceptually determine the value of the chemical potential that  $\Omega_T(\rho)$  favors. By the symmetry of  $\Omega_T(\rho)$  we can restrict ourselves to the positive chemical potential without loss of generality. At first, consider the case of zero temperature. According to Fermi-Dirac statistics, fermions fill all the energy levels to the Fermi energy (=chemical potential) and antifermions are suppressed. Hence, the larger chemical potential, the larger charge (number) density (= fermion number density–antifermion number density). This result is retained at nonzero temperature. Since the grand free energy density is minus the pressure [20], it is a decreasing function of the (positive) charge density. Consequently, the large charge density (i.e., large chemical potential) is preferred and, for all temperatures,  $\Omega_T(\rho)$  is minimized at large chemical potential ( $|\rho| \rightarrow \infty$ ). This implies SNR for the charge conjugation symmetry in the open system of free massless Dirac fermions. Moreover, we can guess the functional form of  $\Omega_T(\rho)$  by dimensional analysis: At zero temperature,  $\Omega_0(\rho) \propto -|\rho|^d$  in  $d$  dimensions. At sufficiently high temperature,  $\Omega_T(\rho) \propto -T\rho^2 + O(\rho^4/T)$  in three

dimensions and  $\Omega_T(\rho) \propto -\rho^2 + O(\rho^4/T^2)$  in two dimensions, up to a field-independent constant. These qualitative results can be checked explicitly from Eqs.(15), (19), (20) and (24).

Now let us consider the contribution  $P_R(\rho)$  from the probability distribution of the random chemical potential. In three dimensions, for large  $|\rho|$ ,  $\Omega_T(\rho)$  exceeds  $P_R(\rho)$  at zero and sufficiently high temperature, irrespective of the value of the disorder strength. Therefore, the initially broken charge conjugation symmetry is not restored at high temperature. In two dimensions  $P_R(\rho)$  is comparable to  $\Omega_T(\rho)$ . The probability distribution of the random chemical potential for weak disorder (small  $R^2$ ) is dominated at  $\rho = 0$  and the charge conjugation symmetry is preserved at zero and high temperature. However, for strong disorder (large  $R^2$ ) all values of the chemical potential have small probability density and so the fate of the charge conjugation symmetry is determined by  $\Omega_T(\rho)$ . Thus, in this case, the initially broken charge conjugation symmetry is not restored at high temperature.

## V. CONCLUSIONS

In the present paper we examined the symmetry behavior of the Gross-Neveu model with random chemical potential which is equivalent to the four-fermion model Eq.(3). We used the leading approximation in the  $1/N$  expansion (i.e., the mean field approximation). Our model has the charge conjugation symmetry as well as the  $Z_2$  chiral symmetry. The initially broken  $Z_2$  chiral symmetry is always restored at high temperature. In three dimensions, the charge conjugation symmetry that is broken at zero temperature, is not restored at high temperature, irrespective of the value of the disorder strength  $R^2$ . In two dimensions, at zero temperature, the charge conjugation symmetry is not broken for weak disorder ( $0 < R^2 < R_c^2(= \pi/2)$ ), but broken for strong disorder ( $R^2 > R_c^2$ ). Therefore, our system exhibits a quantum phase transition at  $R^2 = R_c^2$  as the value of  $R^2$  is varied. For any given value of  $R^2$  the high-temperature behavior of the charge conjugation symmetry is the same as its zero-temperature behavior. Hence the charge conjugation symmetry remains broken at

high temperature (i.e., symmetry nonrestoration) for  $R^2 > R_c^2$ . By examining the existence of the kink solutions we checked that the mean field approximation is reliable even in two dimensions at zero and high temperature.

In addition, we discussed our results on charge conjugation symmetry nonrestoration conceptually, after neglecting the four-fermion interaction  $(\bar{\Psi}\Psi)^2$  for convenience because symmetry nonrestoration is the effect of the random chemical potential. The behavior of the charge conjugation symmetry is determined by the competition of two terms in the finite-temperature effective potential: (i) the term  $\rho^2/(2R^2)$  from the Gaussian distribution for the chemical potential  $\rho$  that favors the symmetric phase ( $\langle\rho\rangle = 0$ ) and (ii) the grand free energy for free massless Dirac fermions which favors the broken phase ( $|\langle\rho\rangle| \rightarrow \infty$ ).

As further work, it would be worthwhile to perform next-to-leading order calculations and consider a non-Gaussian distribution for the random chemical potential.

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## FIGURES

FIG. 1. The zero-temperature effective potential  $V_0/(NM^3)$  in three dimensions as a function of  $\sigma/M$  and  $\rho/M$  for  $R^2M = 10$ , in the case of the  $Z_2$  chiral symmetry breaking ( $G^2M = -\pi$ ).

FIG. 2. The zero-temperature effective potential  $V_0/(NM^2)$  in two dimensions as a function of  $\sigma/M$  and  $\rho/M$ , for (a)  $R^2 = 1$  and (b)  $R^2 = 10$ .



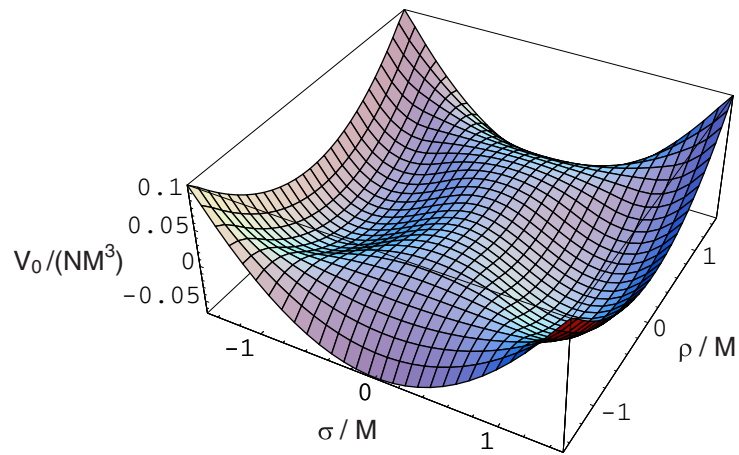


FIG. 1. S. I. Hong and J. B. Kogut

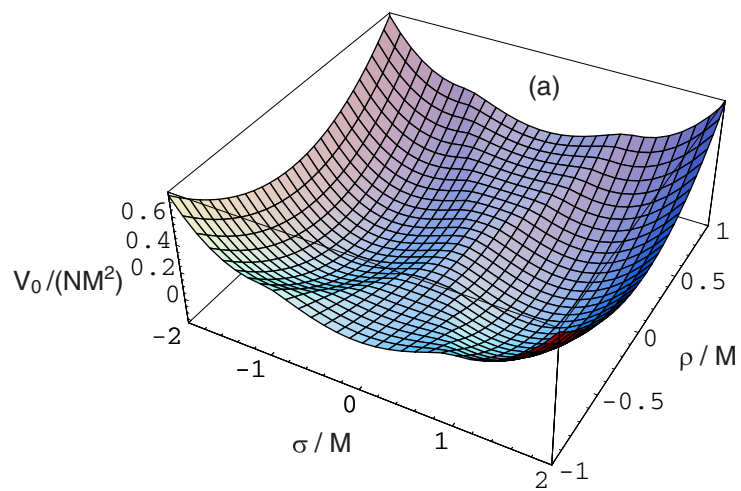


FIG. 2 (a). S. I. Hong and J. B. Kogut

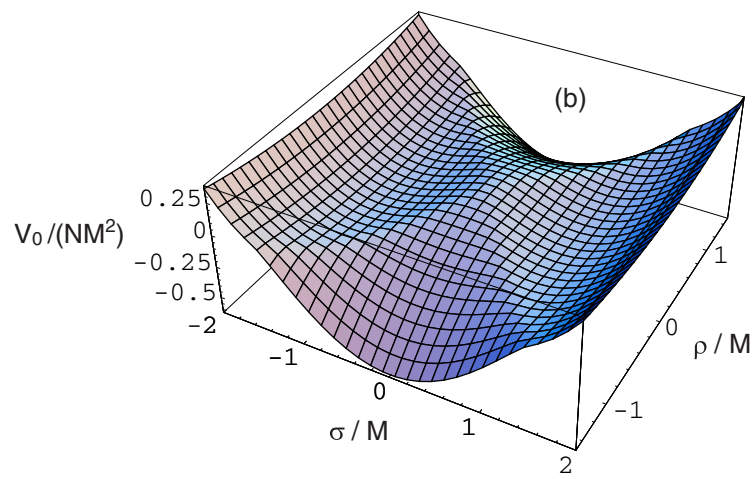


FIG. 2 (b). S. I. Hong and J. B. Kogut